MST121

Assignment Booklet I 2006B

Contents **Cut-off date**

Submission instructions for TMAs and CMAs

Please send all your answers to each tutor-marked assignment (TMA), together with a completed assignment form (PT3), to reach your tutor on or before the cut-off date shown above. **TMA 01 Part 2 is an exception to this; please see the submission instructions for TMA 01 on page 3.**

You will find instructions on how to fill in the PT3 form in the *Guide to Preparation*. Remember to fill in the correct assignment number and allow sufficient time in the post for each assignment to reach its destination on or before the cut-off date. Remember also to pay the correct postage charge to cover the weight of each assignment and its envelope.

The completed computer-marked assignment (CMA) form should be posted in the envelope provided to arrive at Walton Hall by the cut-off date. Detailed instructions on how to use the form are given in the *Guide to Preparation*.

You are advised to keep a copy of your answers to each assignment in case of loss in the mail. Also keep all your marked assignments as you may want to make reference to them later.

Plagiarism

The University considers plagiarism to be a serious matter, and therefore we draw your attention to the appendix on plagiarism in the *Assessment Handbook* which is entitled 'What constitutes plagiarism or cheating?'. Please note that references to 'assignments' should be taken to include any piece of work submitted for assessment, not just tutor-marked assignments.

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Points to note when preparing solutions to TMA questions

- Contact your tutor if the meaning of any part of a question does not seem clear.
- Your solutions should not involve the use of Mathcad, except in those parts of questions where this is explicitly required or suggested.
- Where a question involves mathematical calculation, show all your working. You may not receive full marks for a correct final answer that is not supported by working. You may receive some marks for working even if your final answer is incorrect or your solution is incomplete.
- Whenever you perform a calculation using a numerical answer found earlier, you should use the full calculator-accuracy version of the earlier answer to avoid rounding errors.
- Number all of your pages, including any computer printouts.
- Indicate in each solution the page numbers of any computer printouts associated with that solution.
- The marks allocated to the parts of the questions are indicated in brackets in the margin. Each TMA is marked out of 100. Your overall score for a TMA will be the sum of your marks for each question part.

TMA MST121 01 Part 1 Cut-off date 22 February 2006

Special submission instructions for TMA MST121 01

TMA MST121 01 is in two parts. Part 1 comprises Questions 1–3, on Chapters A0 and A1. Part 2 comprises Questions 4–7, on Chapters A2 and A3, and follows immediately after Part 1.

Part 1 is marked out of 40; the whole TMA is marked out of 100.

Please send your answers to Part 1 to your tutor, with an assignment form (PT3). Be sure to fill in the assignment number on this form as

$\boxed{\text{MST121 01}}$.

Your tutor will mark and comment on your solutions to Questions 1–3, and will send your script back to you directly to give you some early feedback on the course. The PT3 form relates to the whole assignment, so it will not be returned to you at that stage, but will be retained by your tutor so that your marks for Part 2 of the assignment may be entered on it. The form will then be sent to you, with your marked solutions to Part 2, via Walton Hall so that your marks can be recorded.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter A0.

(a) Solve the following pair of simultaneous equations algebraically. Any non-integer answers should be given as fractions in their simplest form.

$$5a + 3b = 9$$
$$7a + 12b = 10$$
[4]

(b) Rearrange the following equation to find x in terms of y and t, in simplest form. (Assume that $2t - y \neq 0$.)

$$2t(x - 2t) = y(x - y)$$
[4]

(c) Solve the equation below for x.

$$\frac{x-2}{x} - \frac{7}{3x+2} = 0$$
[4]

In parts (d) and (e) you are asked to use Mathcad. You may present the printouts for parts (d) and (e) on a single page.

(d) Create a new Mathcad worksheet, and define a variable p to have value 2.84. Type in the expression below, and evaluate it for the given value of p.

$$4p^3 - 7p^2 + 6p + 3$$

Now choose **Result...** from the **Format** menu and alter the Number Format to display your answer correct to 5 decimal places.

You should provide a printout of your working. [4]

(e) Use Mathcad to enter and then factorise the expression below.

$$147x^3 - 217x^2 - 168x + 28$$

You should provide a printout of your working. [4]

Question 2 – 15 marks

You should be able to answer this question after studying Chapter A1.

(a) Consider the sequence given by

$$u_1 = 4.7, \quad u_{n+1} = u_n + 1.6 \quad (n = 1, 2, 3, \ldots).$$

(i) State what type of sequence this is. [*1*]

(ii) Write down the first four terms of the sequence. [*2*]

(iii) Find a closed form for the sequence. [*3*]

(iv) Use the closed form from part (a)(iii) to find the value of n when $u_n = 121.5$. [*2*]

(b) Consider the following geometric sequence.

$$8, \quad 10, \quad 12.5, \quad 15.625, \quad \ldots$$

(i) Write down a recurrence system that describes this sequence. (Denote the sequence by x_n, and its first term by $x_1 = 8$.) [*3*]

(ii) Find a closed form for this sequence. [*2*]

(iii) Use the closed form from part (b)(ii) to find the 10th term of the sequence, giving your answer correct to 4 decimal places. [*2*]

Question 3 – 5 marks

You should be able to answer this question after studying Chapter A1.

Consider the linear recurrence sequence

$$x_1 = 12, \quad x_{n+1} = 0.7x_n - 3 \quad (n = 1, 2, 3, \ldots).$$

(a) Find a closed form for the sequence. [*2*]

(b) Use the closed form to find the 8th term of the sequence, correct to 4 significant figures. [*1*]

(c) Describe the long-term behaviour of the sequence. [*2*]

This part of the TMA covers Chapters A2 and A3.

Remember that you should NOT submit a second PT3 form with this part of the assignment.

Question 4 – 20 marks

You should be able to answer this question after studying Chapter A2.

This question concerns the three points $A(6, 3)$, $B(2, -5)$ and $C(8, -3)$ and the circle that passes through them.

(a) (i) Find the slope of the line that passes through A and B. [2]

(ii) Find the midpoint of the line segment AB. [2]

(iii) Find the equation of the line corresponding to the parametric equations

$$x = t - 3, \quad y = -\tfrac{1}{2}t + \tfrac{5}{2}.$$ [2]

(iv) Show that the perpendicular bisector of AB is the same line as in part (iii) above. [2]

(b) Find the equation of the perpendicular bisector of the line segment AC. [4]

(c) (i) From your answers to parts (a)(iv) and (b), find the centre of the circle that passes through A, B and C. [4]

(ii) Find the radius of this circle, giving your answer correct to 2 decimal places. [2]

(iii) Write down the equation of this circle. (There is no need to simplify your answer.) [2]

(As a check, the coordinates of each of the points A, B and C should satisfy this equation.)

Question 5 – 10 marks

You should be able to answer this question after studying Chapter A2.

(a) The equation

$$x^2 + 6x + y^2 - 2y - 15 = 0$$

represents a circle. Find its centre and radius. [6]

(b) Find any points at which the circle in part (a) intersects the line $y = -x - 1$. [4]

Question 6 – 20 marks

You should be able to answer this question after studying Chapter A3.

(a) This part of the question concerns the parabola which is the graph of the function

$$f(x) = \tfrac{1}{2}(x-3)^2 - 18.$$

(i) Explain how the graph of the parabola can be obtained from the graph of $y = x^2$ by using appropriate translations and scalings. [4]

(ii) Using your answer to part (i), or otherwise, write down the coordinates of the vertex of the parabola. [1]

(iii) Find the x-intercepts and the y-intercept of the parabola. [4]

(iv) Sketch the parabola, marking on your sketch the coordinates of the vertex and of the points at which the graph crosses the axes. [3]

You should not use Mathcad to produce your sketch.

(v) What is the image set of the function f? (You should express your answer in interval notation.) [2]

(b) This part of the question concerns the function

$$g(x) = \tfrac{1}{2}(x-3)^2 - 18 \quad (5 \le x \le 9).$$

(The function g has the same rule as the function f in part (a), but a smaller domain.)

(i) Use your sketch from part (a)(iv) to explain why g has an inverse function g^{-1}. [1]

(ii) Specify the domain and image set of g^{-1}, and find its rule. [4]

(iii) Sketch the graph of $y = g^{-1}(x)$. [1]

Question 7 – 10 marks

You should be able to answer this question after studying Chapter A3.

(a) Solve the equation $9 = 7^x$ by applying ln (the natural logarithm function) to both sides. Give your answer correct to 5 decimal places. [2]

(b) Check your answer to part (a) by using a Mathcad 'solve block' to solve the equation $9 = 7^x$ as it stands, obtaining a numerical solution using the methods covered in Mathcad file 121A3-02. Give your answer correct to 5 decimal places.

Provide a printout of your solution. [3]

You may need to insert the definition $x := x$ in your worksheet before doing part (c). (For more details, see the bottom of page 49 in A Guide to Mathcad.*)*

(c) Check your answer to part (a) by first using Mathcad to solve the equation $9 = 7^x$ as it stands symbolically, and then converting the exact symbolic answer into a numerical solution, using the methods covered in Mathcad file 121A3-03. Give your numerical answer correct to 5 decimal places.

Provide a printout of your solution. [2]

(d) Use Mathcad to plot the graph of the function

$$f(x) = 7^x - 9 \quad (-1 \le x \le 2).$$

Provide a printout of your graph.

(You might like to start from Mathcad file 121A3-04.) [3]

This assignment covers Block B.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter B1.

A population of warthogs was introduced into a reserve on a particular date, and the approximate size of the population was determined on the same date in each subsequent year. The size of the initial population was 50, and it had grown to approximately 125 after one year. After another year, the size of the population was approximately 250, and the following year it was approximately 285. Assume that the behaviour of the population satisfies the logistic model.

(a) Show that the annual proportionate growth rate for the population of size 50 was approximately 1.5, and that the annual proportionate growth rate for the population of size 250 was approximately 0.14. [3]

(b) Find the corresponding values of the annual proportionate growth rate, r, for low population levels, and the equilibrium population level, E. [7]

You should use Mathcad file 121B1-01 in parts (c) and (d) below.

(c) By adjusting the values of variables in a copy of the worksheet in Mathcad file 121B1-01, produce a Mathcad graph showing the behaviour of the population over the first 25 years, as predicted by the model.

You should provide a printout of page 2 of the worksheet as amended. [4]

(d) According to the model, how many years does it take for the size of the population to reach a value within 2.5% of its equilibrium level? [4]

(e) Describe the long-term behaviour of the population as predicted by the model, justifying your answer without reference to Mathcad. [2]

Question 2 – 15 marks

You should be able to answer this question after studying Chapter B1.

(a) (i) Find the sum of the integers from 23 to 101 inclusive. [3]

(ii) Hence find the value of $\displaystyle\sum_{i=23}^{101}(5+7i)$. [3]

(b) For each of the sequences below, decide whether it converges and, if it does, state its limit. Justify your answers briefly.

(i) $a_n = \dfrac{3+12n^4}{3-4n^2} \quad (n = 1, 2, 3, \ldots)$ [4]

(ii) $b_n = \dfrac{8(0.2)^n + 0.5}{0.9 - 8(0.8)^n} \quad (n = 1, 2, 3, \ldots)$ [2]

(c) Find the sum of the infinite series

$$\left(\tfrac{1}{7}\right)^4 + \left(\tfrac{1}{7}\right)^5 + \left(\tfrac{1}{7}\right)^6 + \left(\tfrac{1}{7}\right)^7 + \cdots.$$ [3]

Question 3 – 15 marks

You should be able to answer this question after studying Chapter B2.

This question concerns the following matrices:

$$\mathbf{A} = \begin{pmatrix} -4 & -2 \\ 6 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 7 & 3 \\ -1 & -1 \\ -2 & 9 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}.$$

(a) Evaluate each of the following, where possible. (Give sufficient details of your working to make it clear that you have not needed to use Mathcad.) Where evaluation is not possible, explain why not.

(i) $3\mathbf{A} - 3\mathbf{B}$

(ii) $2\mathbf{A} - 5\mathbf{C}$

(iii) $\mathbf{BC}$

(iv) $\mathbf{CB}$

(v) $\mathbf{A}^{-1}$

(vi) $\mathbf{C}^{-1}$ [11]

(b) Use matrices to solve the following system of linear equations.

$$5x - y = 3$$
$$4x + y = 6$$ [4]

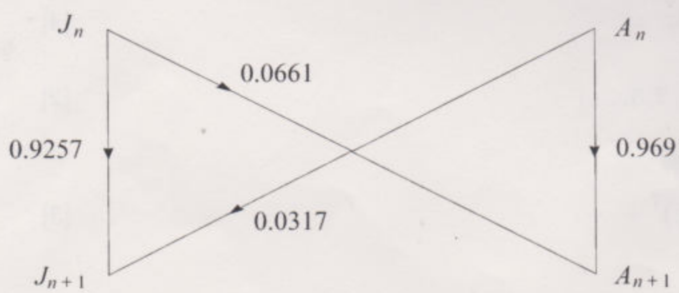
Question 4 – 20 marks

You should be able to answer this question after studying Chapter B2.

The following table gives data for a fictitious country in the year 1800.

	Juveniles (aged under 15 years)	Adults (aged 15 years and over)
Subpopulation size (in millions)	1.2	3.7
Birth rate	0	0.0317
Death rate	0.0082	0.0309

The diagram below shows a network model of the changes in the subpopulations from year to year. Here J_n and A_n denote the sizes (in millions) of the subpopulations of juveniles and adults, respectively, n years after the year 1800. It has been assumed that, in any year, $\frac{1}{15}$ of the juveniles become adults.



(a) Explain why the number labelling the connection from J_n to A_{n+1} is 0.0661, and why the number labelling the connection from J_n to J_{n+1} is 0.9257. [4]

(b) Write the network model as a matrix equation. [2]

For parts (c) and (d) you will need to use Mathcad file 121B2-02. For each of these parts, provide a printout of page 2 of the amended worksheet, showing your work. For part (c)(iii), provide printouts of any pages of the worksheet that show amendments that you have made, or results that you have used, for this part.

(c) (i) Edit the matrix $\mathbf{M}$, and the vector whose entries are the initial subpopulation sizes J_0 and A_0, in a copy of the worksheet in Mathcad file 121B2-02, so that the worksheet shows the predicted changes in population for the country considered in this question. [*2*]

(ii) Find the sizes of the subpopulations predicted by the model for the years 1815 and 1830, giving your answers to the nearest thousand. [*2*]

(iii) Calculate the ratio of juveniles to adults in the long term, correct to 3 decimal places. [*4*]

(d) Suppose now that there is an annual net immigration to the country of 0.018 million adults. The matrix equation for a revised model, taking account of this immigration, is

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \mathbf{M}\begin{pmatrix} J_n \\ A_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0.018 \end{pmatrix},$$

where $\mathbf{M}$ is the same matrix as in part (b).

(i) The equation from part (b) appears as a Mathcad formula on page 2 of the worksheet. Edit this formula to make it the equation for the revised model. (To do this, you should first select the entire right-hand side of the formula and type '+'. Then you can use the 'Matrix' toolbar to create a 2×1 matrix, and enter the appropriate values in the placeholders.) [*4*]

(ii) Find the sizes of the subpopulations predicted by the revised model for the years 1815 and 1830, giving your answers to the nearest thousand. [*2*]

Question 5 – 15 marks

You should be able to answer this question after studying Chapter B3.

You may find it helpful to draw a diagram for this question.

A bird has a flight speed in still air of $8.9\,\mathrm{m\,s^{-1}}$. It is pointed in the direction N 33°E, but flies in a wind of speed $14.7\,\mathrm{m\,s^{-1}}$ *from* the direction S 72°E. Take $\mathbf{i}$ to be $1\,\mathrm{m\,s^{-1}}$ due east and $\mathbf{j}$ to be $1\,\mathrm{m\,s^{-1}}$ due north. Also, take

$\mathbf{v}_\mathrm{b}$ to be the velocity of the bird in still air,

$\mathbf{v}_\mathrm{w}$ to be the velocity of the wind,

$\mathbf{v}$ to be the resultant velocity of the bird.

(a) Express each of the vectors $\mathbf{v}_\mathrm{b}$ and $\mathbf{v}_\mathrm{w}$ in component form, giving the components correct to 4 decimal places. [*5*]

(b) Hence show that the resultant velocity $\mathbf{v}$ of the bird is given in component form approximately by

$$\mathbf{v} = -9.1332\,\mathbf{i} + 12.0067\,\mathbf{j}.$$ [*2*]

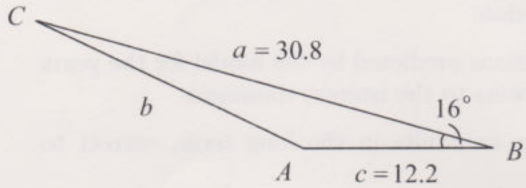
(c) Find the overall speed $|\mathbf{v}|$ of the bird (to 4 significant figures), and its direction of travel as a bearing (with the angle correct to 1 decimal place). [*5*]

(d) The bird begins its flight from a point on the south bank of a river that flows due west and is 260 metres wide. How long does it take the bird to cross the river, and what is the distance that it has travelled in this time? Give your answers to 3 significant figures. [*3*]

Question 6 – 15 marks

You should be able to answer this question after studying Chapter B3.

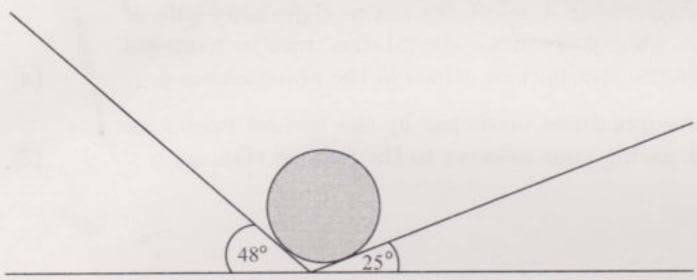
(a) The triangle ABC, labelled according to the convention shown in Figure 3.1(a) on page 30 of Chapter B3, has angle $B = 16°$ and side lengths $a = 30.8$ and $c = 12.2$, as shown below.



(i) Use the Cosine Rule to find the side length b, correct to 2 decimal places. [2]

(ii) Find the angles A and C, correct to 1 decimal place. [3]

(b) A ball of mass 8.9 kg rests in the corner between two sloping surfaces, as shown in the diagram below. The surfaces make angles of 48° and 25° with the horizontal. Assume that the only forces acting on the ball are its weight and the normal reactions from the two surfaces. Take the magnitude of the acceleration due to gravity to be $g = 10\,\mathrm{m\,s^{-2}}$.



(i) Draw a force diagram for the forces acting on the ball, indicating the angles between the forces, and defining any symbols that you use to denote the forces. [3]

(ii) Draw a corresponding triangle of forces. [3]

(iii) Use the triangle of forces to find the magnitudes of the normal reactions from the two surfaces, to 3 significant figures. [4]

This assignment covers Block C.

Question 1 – 20 marks

You should be able to answer this question after studying Chapter C1.

This question concerns the function

$$f(x) = x^3 + 9x^2 + 15x - 13.$$

(a) Find the stationary points of this function. [6]

(b) (i) Using the strategy to apply the First Derivative Test, classify the left-hand stationary point found in part (a). [4]

(ii) Using the Second Derivative Test, classify the right-hand stationary point found in part (a). [3]

(c) Find the y-coordinate of each of the stationary points on the graph of the function $f(x)$, and also evaluate $f(0)$. [3]

(d) Hence draw a rough sketch of the graph of the function $f(x)$. [4]

Question 2 – 15 marks

You should be able to answer this question after studying Chapter C1.

In each of the following parts, simplify your answers where appropriate.

(a) (i) Write down the derivative of each of the functions

$$f(x) = e^{-2x} \qquad \text{and} \qquad g(x) = \sin(7x).$$ [2]

(ii) Hence, by using the Product Rule, differentiate the function

$$k(x) = e^{-2x}\sin(7x).$$ [2]

(b) (i) Write down the derivative of each of the functions

$$f(t) = \ln(5t) \quad (t > 0) \qquad \text{and} \qquad g(t) = t^3 + 1.$$ [2]

(ii) Hence, by using the Quotient Rule, differentiate the function

$$k(t) = \frac{\ln(5t)}{t^3 + 1} \quad (t > 0).$$ [3]

(c) (i) Write down the derivative of each of the functions

$$f(x) = \cos(\tfrac{1}{2}x) \qquad \text{and} \qquad g(u) = u^{1/3} \quad (u > 0).$$ [2]

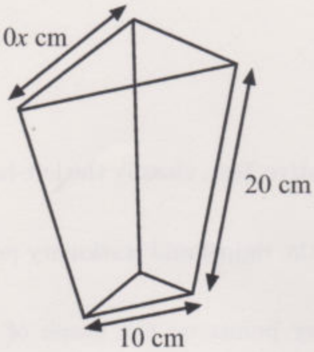
(ii) Hence, by using the Composite Rule, differentiate the function

$$k(x) = \left(\cos(\tfrac{1}{2}x)\right)^{1/3} \quad (-\pi < x < \pi).$$ [4]

Question 3 – 10 marks

You should be able to answer this question after studying Chapter C1.

An open container is such that each horizontal cross-section is an equilateral triangle. Its base has sides of length 10 cm, and its top has sides of length $10x$ cm. Each sloping edge has length 20 cm. The surface of the container is modelled by part of an inverted triangular pyramid, as shown below.



The capacity $V(x)$ litres of the container is given by

$$V(x) = \tfrac{1}{12}(x^2 + x + 1)\sqrt{12 - (x-1)^2} \quad (0 \le x \le 1 + 2\sqrt{3}).$$

(You are *not* asked to derive this formula. Note that 1 litre $= 1000\,\text{cm}^3$.)

(a) Explain why the condition $0 \le x \le 1 + 2\sqrt{3}$ is placed on the applicability of the formula for $V(x)$. 4·64 [*1*]

For parts (b) and (c) (and for part (d), if you use Mathcad there) you should provide a printout annotated with enough explanation to make it clear what you have done.

NB: If you define x to be a range variable in part (b) and wish to use x in a symbolic calculation in part (c), then you will need to insert the definition $x := x$ between the two parts in your worksheet. (For more details, see the bottom of page 49 in A Guide to Mathcad.*)*

(b) Use Mathcad to obtain the graph of the function $V(x)$. [*2*]

(c) This part of the question requires the use of Mathcad in each sub-part.

(i) By using the differentiation facility and the symbolic keyword 'simplify', show that any root of the equation $V'(x) = 0$ is also a root of the cubic equation

$$x^3 - x^2 - 8x - 4 = 0.$$ [*2*]

(ii) By applying a 'solve block' to this cubic equation, or otherwise, find a value of x for which $V'(x) = 0$. [*2*]

(iii) Verify, by the Second Derivative Test, that this value of x corresponds to a local maximum of $V(x)$. (It should be apparent from the graph obtained in part (b) that this is also an overall maximum within the domain of $V(x)$.) [*2*]

(d) Using Mathcad, or otherwise, find the maximum possible capacity of the container, according to the model. [*1*]

Question 4 – 25 marks

You should be able to answer this question after studying Chapter C2.

(a) Find the indefinite integrals of the following functions.

(i) $f(t) = 2\cos(4t) - 3e^{5t}$ [3]

(ii) $g(x) = \dfrac{19 + 15x^3}{x} \quad (x > 0)$ [4]

(iii) $h(u) = \sin^2(\frac{1}{10}u)$ [6]

(b) Evaluate $\displaystyle\int_2^4 6x(2x^2 - 1)\,dx$. [6]

(c) (i) Write down a definite integral that will give the value of the area under the curve $y = x^3\cos(\frac{1}{4}x)$ between $x = \pi$ and $x = 2\pi$. [2]
(You are *not* asked to evaluate the integral by hand.)

(ii) Use Mathcad to find the area described in part (c)(i), giving your answer correct to 3 decimal places.
You should provide a printout of your working. [4]

Question 5 – 10 marks

You should be able to answer this question after studying Chapter C2.

A rocket is modelled by a particle which moves along a vertical line. From launch, the rocket rises until its motor cuts out after 22 seconds. At this time it has reached a height of 440 metres above the launch pad, and attained an upward velocity of $40\,\mathrm{m\,s^{-1}}$. From this time on, the rocket has constant upward acceleration $-10\,\mathrm{m\,s^{-2}}$ (due to the effect of gravity alone).

Choose the s-axis (for the position of the particle that represents the rocket) to point upwards, with origin at the launch pad. Take $t = 0$ to be the time when the rocket motor cuts out.

(a) What is the maximum height (above the launch pad) reached by the rocket? [4]

(b) How long (from launch) does the rocket take to reach this maximum height? [2]

(c) After how long (from launch) does the rocket crash onto the launch pad? [4]

Question 6 – 20 marks

You should be able to answer this question after studying Chapter C3.

(a) Find the solution of the initial-value problem

$$\frac{dy}{dx} = \frac{\sin(3x)}{2 + \cos(3x)}, \quad y = 4 \text{ when } x = 0.$$

(You may find equation (2.4) in Chapter C2 helpful when integrating.) [6]

(b) (i) Find, in implicit form, the general solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^{1/2}(e^{-x} - e^{x})}{(e^{x} + e^{-x})^2} \quad (y > 0).$$

(You may find equation (2.3) in Chapter C2 helpful when integrating.) [7]

(ii) Find the corresponding explicit form of this general solution. [2]

(iii) Find the corresponding particular solution that satisfies the initial condition $y = 1$ when $x = 0$. [3]

(iv) What is the value of y given by this particular solution when $x = 0.5$? Give your answer correct to 4 decimal places. [2]

This assignment covers Block D.

You will need to use OUStats to answer Questions 11, 12 and 14.

Questions 1 to 9 are on Chapter D1.

Questions 1 to 3

A standard pack of 52 playing cards consists of four suits. Each suit contains 13 cards: ace, jack, queen, king and the 'number cards' with the numbers $2, 3, \ldots, 10$. A pack of cards is shuffled, and the top card is turned face up. This card is returned to the pack. The pack is then shuffled a second time, and the top card is again turned face up.

1 Choose the option that is the probability that the first card turned face up is not a number card.

2 Choose the option that is the probability that the second card turned face up is a number card displaying an even number.

Options for Questions 1 and 2

A $\frac{1}{13}$ **B** $\frac{5}{52}$ **C** $\frac{9}{52}$ **D** $\frac{3}{13}$

E $\frac{4}{13}$ **F** $\frac{5}{13}$ **G** $\frac{9}{13}$ **H** $\frac{12}{13}$

3 Choose the option that is the probability that the first card turned face up is a king and the second is not a king.

Options for Question 3

A $\frac{1}{13}$ **B** $\frac{2}{13}$ **C** $\frac{1}{169}$ **D** $\frac{12}{169}$ **E** $\frac{144}{169}$ **F** $\frac{51}{2704}$

Questions 4 and 5

Three regular tetrahedral dice each have faces labelled 1, 2, 3 and 4. The three dice are rolled together.

4 Choose the option that is the probability that the first die lands on the face labelled 3, and both the second and third dice land on even numbers.

5 Choose the option that is the probability that at least one of the dice lands on a 2.

Options for Questions 4 and 5

A $\frac{1}{16}$ **B** $\frac{1}{4}$ **C** $\frac{27}{64}$ **D** $\frac{7}{16}$

E $\frac{9}{16}$ **F** $\frac{37}{64}$ **G** $\frac{3}{4}$ **H** $\frac{15}{16}$

Questions 6 and 7

Anna likes to arrive at work by 8.30 am but, because of traffic, she is sometimes late. On any one day, the probability that she will be late is $\frac{1}{5}$. Whether she is late on any particular day is independent of whether she was late on any other day. Anna works 5 days per week, from Monday to Friday.

6 Choose the option that is closest to the probability that, during a given week, Anna arrives at work by 8.30 am on Monday to Thursday, but is late on Friday.

Options for Question 6

A	0.0013	**B**	0.0051	**C**	0.0064
D	0.0819	**E**	0.1024	**F**	0.4096

7 Choose the option that is closest to the probability that Anna arrives at work by 8.30 am every day in a given working week.

Options for Question 7

A	0.0003	**B**	0.0655	**C**	0.2621
D	0.3277	**E**	0.6723	**F**	0.9997

Question 8

In a board game, two dice are rolled at each turn. Choose the option that is the number of turns, on average, that a player can expect to take in order to obtain a double six.

Options

A 3 **B** 6 **C** 12 **D** 18 **E** 36 **F** 72

Question 9

A breakfast cereal manufacturer is giving away a free toy in each packet. There are four different types of toy to collect, but there is no way of knowing which type is inside any particular packet without opening the packet. Each type of toy is equally likely to be found. Choose the option that is closest to the number of packets, on average, that you would need to open in order to obtain two different types of toy.

Options

A	0.33	**B**	0.75	**C**	1.33
D	1.75	**E**	2.33	**F**	2.75

Questions 10 to 12 are on Chapter D2.

Question 10

The ages, in months, at which five children began to walk are given below.

11 14 13 18 15

Choose the option that is closest to the sample standard deviation.

Options

A	1.29	**B**	2.32	**C**	2.59
D	5.36	**E**	6.70	**F**	13.40

Questions 11 and 12

You should use OUStats for these questions.

The variation in examination marks for a particular course may be modelled by a normal distribution with mean 57 and standard deviation 12.

11 Choose the option that is closest to the mark above which approximately 10% of examination marks will lie.

Options for Question 11

A	37.3	**B**	41.6	**C**	46.9
D	67.1	**E**	72.4	**F**	76.7

12 Choose the option that is closest to a range of values, symmetrical about the mean, within which approximately 95% of examination marks will lie.

Options for Question 12

A	$(23.3, 90.7)$	**B**	$(33.5, 74.3)$	**C**	$(33.5, 80.5)$
D	$(37.3, 74.3)$	**E**	$(37.3, 76.7)$	**F**	$(41.6, 72.4)$

Questions 13 to 16 are on Chapter D3.

Questions 13 to 15

The distribution of the weights of the contents of boxes of a certain breakfast cereal, labelled as containing 350 g, has mean 351.5 g and standard deviation 6 g.

13 Choose the option that is closest to the standard error of the mean contents (in grams) for samples of 35 cereal boxes.

Options for Question 13

A	0.070	**B**	0.171	**C**	0.414
D	0.986	**E**	1.014	**F**	6

The sampling distribution of the mean weight of the contents, for samples of 30 cereal boxes, has mean 351.5 g and standard deviation 1.095 g.

You should use OUStats for Question 14.

14 Choose the option that is closest to the probability that the mean weight of the contents of a sample of 30 cereal boxes will be less than 350 g.

Options for Question 14

A	0.00	**B**	0.09	**C**	0.16
D	0.40	**E**	0.60	**F**	0.91

15 Choose the option that gives a range of values within which the mean weight of the contents of approximately 90% of samples of 30 cereal boxes will lie.

Options for Question 15

A	$(341.7, 361.3)$	**B**	$(349.4, 353.6)$	**C**	$(349.7, 353.3)$
D	$(351.1, 351.9)$	**E**	$(351.2, 351.8)$	**F**	$(351.3, 351.6)$

Question 16

Nutritionists want to assess the effectiveness of a new diet. To do this, 42 overweight volunteers tried out the diet for one month. The volunteers were all weighed before starting the diet, and after one month on the diet. The sample mean weight loss was 3.62 kg, and the sample standard deviation was 8.97 kg.

Choose the option that gives an approximate 95% confidence interval for the mean weight loss, in kilograms, for individuals who have been using the diet for one month.

Options

A	$(-13.96, 21.20)$	**B**	$(0.91, 6.33)$	**C**	$(1.35, 5.89)$
D	$(3.20, 4.04)$	**E**	$(7.88, 10.07)$	**F**	$(8.05, 9.89)$

Questions 17 to 24 are on Chapter D4.

Questions 17 to 20

The advertised selling prices (in £) of nine four-bedroom houses in Milton Keynes are given below.

217 500 289 995 263 000 229 995 159 995
279 995 199 950 249 995 299 995

17 Choose the option that is the median advertised selling price (in £).

Options for Question 17

A	229 995	**B**	239 995	**C**	243 380
D	249 995	**E**	256 497.5	**F**	263 000

18 Choose the option that is the lower quartile for the selling prices.

19 Choose the option that is the upper quartile for the selling prices.

Options for Questions 18 and 19

A	199 950	**B**	208 725	**C**	217 500	**D**	223 747.5
E	271 497.5	**F**	279 995	**G**	284 995	**H**	289 995

20 Choose the option that is the range of advertised selling prices.

Options for Question 20

A	62 495	**B**	76 270	**C**	90 045
D	100 045	**E**	130 000	**F**	140 000

Questions 21 and 22

Two secondary schools, A and B, have students of a similar academic standard. Student performance in public examinations for the two schools is also usually very similar.

School A is participating in a project that involves using a new approach to teaching and learning in the subjects English and Mathematics. After the new teaching approach had been used for one year, the students in Year 9 in both schools were given the same examinations in English and Mathematics. In total, 241 students from School A and 172 students from School B took the examinations.

21 The results from the English examination are summarised in the table below.

		Score for English examination	
	Sample size	Sample mean	Sample standard deviation
School A	241	64	12.7
School B	172	60	13.8

Choose the option that is closest to the estimated standard error (ESE) of the difference between the two sample means.

Options for Question 21

A 0.10 **B** 0.13 **C** 0.36

D 1.33 **E** 1.78 **F** 1.87

22 In the Mathematics examination, the sample mean score for students at School A was 62, and the sample mean score for students at School B was 59. The estimated standard error was 1.28.

A two-sample z-test is to be carried out to test whether there is a difference between the mean Mathematics examination scores for Year 9 students who have been taught using the new approach and those who have not. Choose the option that is closest to the value of the test statistic z.

Options for Question 22

A 0.43 **B** 1.83 **C** 2.34

D 2.65 **E** 2.87 **F** 3.00

Questions 23 and 24

A company believes that monthly sales of its products are related to its monthly expenditure on advertising. A straight line is used to model this relationship. The least squares fit line obtained using data for three years has the equation

$$y = 46.5 + 24.8x,$$

where y is the monthly total sales (in £1000s) and x is the monthly total expenditure on advertising (in £1000s).

23 Choose the option that is the estimated total sales (in £) for a month whose advertising expenditure is £20 460.

Options for Question 23

A	553.9	**B**	1458.8	**C**	507 454.5
D	553 908	**E**	1 458 798	**F**	507 454 500

24 Choose the option that is the estimated increase in monthly sales (in £) if there is an increase in monthly advertising expenditure of £3000.

Options for Question 24

A	74.4	**B**	120.9	**C**	164.3
D	74 400	**E**	120 900	**F**	164 300

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